Global stability, analytic scattering, and localized excursions for Maxwell–Higgs fields

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Abstract

Small localized perturbations of a nonzero vacuum scatter into massive physical fields. Near the symmetric equilibrium, arbitrarily small localized scalar data produce a delayed macroscopic excursion and an exact local return.

Introduction

We study the Maxwell–Higgs equations on four-dimensional Minkowski spacetime, with fixed positive parameters e, λ and v.

$$\partial_\mu F^{\mu\nu} = e\,\mathrm{Im}(\bar{\phi} D^\nu \phi)$$

$$D_\mu D^\mu \phi + \lambda(|\phi|^2 - v^2)\phi = 0$$

$$V(\phi) = \frac{\lambda}{4}(|\phi|^2 - v^2)^2$$

Here $D\mu = \partial\mu - ieA\mu$ and $F = dA$.
The nonzero potential changes the physical spectrum and the long-time dynamics.

Two equilibria

Broken vacuum |ϕ| = v

The circle of minima has positive physical masses. Small compatible perturbations remain global and scatter.

Symmetric equilibrium ϕ = 0

The scalar Hessian is negative. Vanishing initial norms can still generate an order-one local excursion.

Selected references

B. E. Gunara, F. T. Akbar, Triyanta, J. S. Kosasih, 2026, to be submitted.
A. Ferdiyan, R. Primulando, F. T. Akbar, and B. E. Gunara, ArXiv 2605.21870.

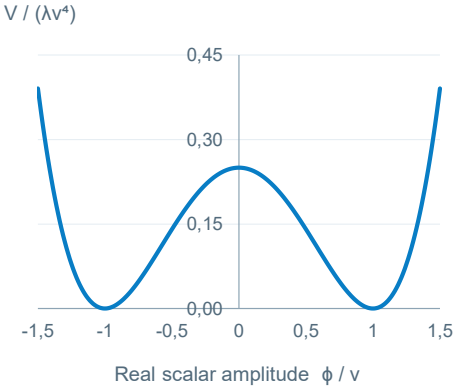
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Research Method

A vacuum-adapted hyperbolic gauge gives six coupled Klein–Gordon equations and propagates the gauge constraint and Gauss's law. Hyperboloidal energies with three commutations yield integrable nonlinear decay. Convergent multilinear expansions give analytic scattering.

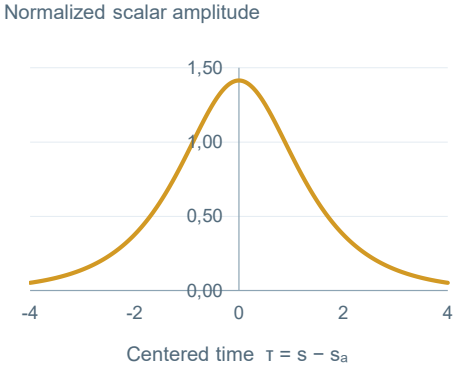
The symmetry-breaking potential



The two minima are stable vacua. The central maximum is the unstable symmetric equilibrium. A real slice of the complex potential.

For the unstable regime, finite propagation embeds a homogeneous Duffing orbit into a large scalar plateau. Relative energy estimates control compatible electromagnetic perturbations through the first return.

The centered excursion



$$H(\tau) = \sqrt{2}\,\mathrm{sech}\,\tau$$

The analytic profile describes the local excursion near its turning time. Curves show the model formulas.

Discussion & Results

1 Stability and analytic scattering

Small smooth compatible data supported in the unit ball admit a unique global solution U with N ≥ 3 commutations. The gauge constraint and Gauss's law hold in the fixed hyperbolic gauge.

$$|U| + |\partial U| \le C\varepsilon(1+|t|)^{-3/2}$$

The gauge-invariant scalar perturbation ρ − v, with ρ = |ϕ|, and vector B = A − e⁻¹d arg ϕ scatter to real Klein–Gordon and divergence-free Proca fields.

$$m_A = ev, \qquad m_h = \sqrt{2\lambda}\,v$$

$$E_{\mathrm{free}}(h_\pm, b_\pm) = E_{\mathrm{physical}}$$

The reduced scattering maps are analytic embeddings. They approximately preserve distances, and their images are analytic graphs.

2 Excursion and local return

$$\|\phi_a(0)\|_{H^{k+1}} \lesssim a\,[\log(1/a)]^{3/2} \longrightarrow 0$$

For scalar plateaus of radius comparable to log(1/a), the initial norm tends to zero. In normalized time, the turning time satisfies $s_a \sim \log(1/a)$.

$$z_a(s_a + \tau, \cdot) = H(\tau) + O(a^2)$$

The O(a²) profile error is sharp at the turning time. On a fixed ball, the scalar Cauchy data return exactly at $2s_a$. Sufficiently small compatible electromagnetic perturbations, including magnetic fields, preserve the first excursion.

The return is local. Only the scalar solution is asserted global.

Conclusion

Massive scattering near |ϕ| = v.
Delayed local excursions near ϕ = 0.