Dominating Partition Dimension of Regular Graphs: Characterization and Computation

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Abstract

The **dominating partition dimension** $\eta_p(G)$ of a connected graph G is the smallest number of classes in a partition of $V(G)$ such that every vertex has its own vector of distances to the classes and a neighbour in another class.

We determine η_p for four families of **cubic graphs**: Möbius ladders M_n, burger graphs with range one $B_\sigma(n,1)$, necklace graphs $N(m)$, and layered cubic graphs G_k. We show that $\eta_p = 4$ for all four families, except for the necklace graphs $N(4)$ and $N(5)$, where the value is 3.

Keyword: dominating partition dimension, partition dimension, resolving partition, cubic graph, Möbius ladder, burger graph, necklace graph.

Introduction

Identifying the vertices of a graph by their distances is a central topic in metric graph theory. Slater (1975) and Harary & Melter (1976) introduced resolving sets and the **metric dimension**, which has applications in robot navigation, network monitoring and chemical structures.

Chartrand, Salehi and Zhang (1998, 2000) replaced single reference vertices by the classes of a partition. This gives the **partition dimension** $\beta_p(G)$.

Hernando, Mora and Pelayo (2019) added a domination condition and introduced the **dominating partition dimension** $\eta_p(G)$. They proved that for every connected graph G,

$$\beta_p(G) \le \eta_p(G) \le \beta_p(G) + 1,$$

and that $\eta_p(Pn) = \eta_p(Cn) = 3$ for $n \ge 3$. Ridwan, Assiyatun and Baskoro (2023) showed that $\beta_p(G) \le \eta_p(G) \le \chi_L(G)$, where χ_L is the locating-chromatic number of a graph G.

The exact values of η_p are still rare, especially for cubic graphs, where symmetry makes it hard to resolve and dominate at the same time.

Objective. To characterize the dominating partition dimension of cubic graphs with a cyclic or layered structure.



Figure 1.Research discussion.

Research Method

All results are proved with the same two-step scheme.

1. **Upper bound, $\eta_p(G) \le 4$.** We build an explicit partition $\Pi = \{S_1, S_2, S_3, S_4\}$ (Figure 2 and Figure 3). We check that every vertex has a neighbour in another class (domination), and that two vertices in the same class always differ in their distance to S_1 or to S_4 (resolvability).

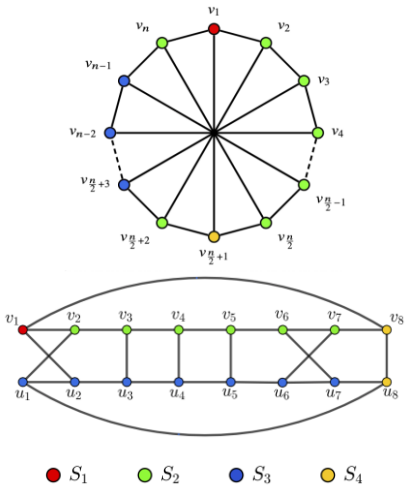

Figure 2. Partition patterns of the Möbius ladder M_n (top) and the burger graph $B_\sigma(8,1)$ (bottom).

2. **Lower bound, $\eta_p(G) \ge 4$.** Suppose $\Pi = \{S_1, S_2, S_3\}$ is a dominating resolving partition. Let p be the largest entry in all representations, reached by a vertex v with $r(v|\Pi) = (0, 1, p)$.
   - *Case* $p = diam(G)$: only a few vertices are this far from v, so S_3 is small. Then at most $2 \cdot diam(G) + c$ vertices can get different representations, which is less than $|V(G)|$.
   - *Case* $p < diam(G)$: vertices that are not adjacent to S_3 can only have the forms (0, 1, *t*) or (1, 0, *t*). Counting them gives $p \le 3$, so at most $6p - 3 \le 15$ representations exist, which is again less than $|V(G)|$.

Both cases give a contradiction, so three classes are never enough. Graphs of small order (for example M_8, M_{10}, M_{12} and $B_\sigma(4,1)$) are checked separately by a direct analysis of distances.

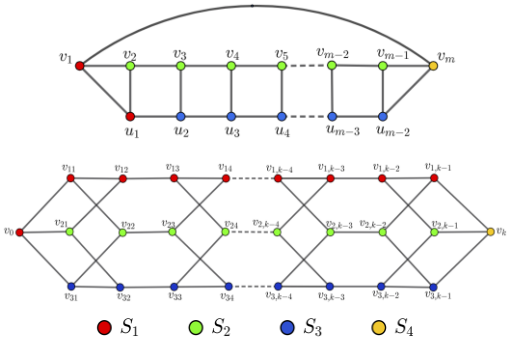

Figure 3. Partition patterns of the necklace graph N(m) (top) and the layered cubic graph G_k (bottom).

Main Results

We determine the values of η_p for four families of cubic graphs.

Theorem 1. Let M_n be a Möbius ladder (the cycle C_n plus edges joining opposite vertices) with even $n \ge 4$. Then $\eta_p(Mn) = 4$. ■

Partition ($n \ge 8$): $S_1 = \{v_1\}$, $S_2 = \{v_2, \ldots, v_{n/2}, v_{n/2+2}, v_n\}$, $S_3 = \{v_{n/2+3}, \ldots, v_{n-1}\}$, $S_4 = \{v_{n/2+1}\}$.

Theorem 2. Let $B_\sigma(n,1)$ be a burger graph with range one and $n \ge 4$. Then $\eta_p(B_\sigma(n,1) = 4$. ■

Partition ($n \ge 5$): $S_1 = \{v_1\}$, $S_2 = \{v_2, \ldots, v_{n-1}\}$, $S_3 = \{u_1, \ldots, u_{n-1}\}$, $S_4 = \{v_n, u_n\}$.

Theorem 3. Let $N(m)$ be a necklace graph (a cycle $v_1 \ldots vm$ and a path $u_1 \ldots u_{m-2}$ linked as in Figure 2) with $m \ge 3$. Then $\eta_p(N(m)) = 3$ if $m \in \{4,5\}$, and $\eta_p(N(m)) = 4$ otherwise. ■

Partition (m ≥ 4): $S_1 = \{v_1, u_1\}$, $S_2 = \{v_2, \ldots, v_{m-1}\}$, $S_3 = \{u_2, \ldots, u_{m-2}\}$, $S_4 = \{v_m\}$.

Theorem 4. Let G_k be a layered cubic graph (levels $L_0, \ldots, L_k$, with edges only between consecutive levels) with odd $k \ge 3$. Then $\eta_p(Gk) = 4$. ■

Partition: $S_1 = \{v_0\} \cup \{v_{1,i}\}$, $S_2 = \{v_{2,i}\}$, $S_3 = \{v_{3,i}\}$, $S_4 = \{v_k\}$, for $1 \le i \le k - 1$.



Figure 4. Presenting the results at IWONT 2026 (top) and JCDCG³ 2026, Japan (bottom).

Conclusion

Möbius ladders, burger graphs with range one, necklace graphs and layered cubic graphs all have **dominating partition dimension 4**, except the necklace graphs N(4) and N(5), which have dominating partition dimension 3. For these families, the domination condition adds at most one class to the partition dimension.

Publications

1. K. Q. Fredlina, P. E. Putri, E. T. Baskoro, On the dominating partition dimension of Möbius ladders and burger graphs, *Electronic Journal of Graph Theory and Applications* 14(2) (2026) 455–469. DOI: 10.5614/ejgta.2026.14.2.13.
2. K. Q. Fredlina, P. E. Putri, E. T. Baskoro, The dominating partition dimension of some classes of cubic graphs, *Indonesian Journal of Combinatorics* (accepted).
3. Y. Hafidh, D.I.D. Primaskun, E.T. Baskoro, On the locating chromatic number of infinite trees, Electron. J. Graph Theory Appl. 14 (2) (2026), 291–302.