# Relasi massa-radius, momen inersia dan tidal love numbers dalam teori gravitasi $f(R,T)$

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## Abstract

Galaxy clusters are the largest virialized structures in the Universe and are predominantly dominated by dark matter. Hydrostatic mass estimates and gravitational lensing measurements frequently differ, creating a discrepancy known as the hydrostatic mass bias. This study derives the hydrostatic mass of galaxy clusters within the framework of Rastall gravity under **two distinct physical scenarios, absence or presence of Dark Matter**. Statistical evaluations confirm that Rastall gravity provides a viable phenomenological framewordk for cluster scalling reactions.

Keyword: Galaxy Cluster, Hydrostatic mass, Rastall gravity, Hydrostatic mass bias, Dark matter.

## Introduction

Galaxy clusters host a hot, diffuse intracluster medium ( $\sim 7\times10^{7}$ K ) emitting thermal X-ray *bremsstrahlung*. Standard cluster mass estimations rely on Newtonian hydrostatic equilibrium assumptions. However, Newtonian hydrostatic masses systematically exceed baryonic mass and differ from gravitational lensing mass estimates. General relativistic (GR) corrections on cluster scales are negligible. **Rastall gravity** (introduced by P. Rastall in 1972) modifies General Relativity by allowing a non-conserved energy-momentum tensor characterized by $\nabla_\mu T^{\mu\nu} = \lambda \nabla^\nu R$, where $\lambda$ parameterizes the deviation from GR ( $\lambda = 0$ ) recovers GR).

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## Research Method

**Field Equation and Relativistic Hydrostatics**: Derived the Rastall-modified Tolman-Oppenheimer-Volkoff (TOV) hydrostatic equilibrium equation for static spherically symmetric metrics:

$$\frac{dp}{dr} = -\frac{\frac{(\rho+p)\left(m+\frac{\kappa}{2}r^3\mathcal{P}\right)}{r(r-2m)}}{1-\frac{\kappa\lambda}{4\kappa\lambda-1}\left(3-\frac{d\rho}{dp}\right)}$$

**Weak-field Limit Formulation**: Evaluated the Newtonian limit under an ideal gas assumption to yield the Rastall hydrostatic mass formula:

$$m(r) = -\frac{k_B}{\mu m_p}\frac{\kappa\lambda-1}{4\kappa\lambda-1}\tau(r)r\left(\frac{d\ln\rho}{d\ln r}+\frac{d\ln\tau}{d\ln r}\right) - \frac{\kappa\lambda}{4\kappa\lambda-1}\left(\frac{\kappa r^3\rho}{2}+\frac{c^2 r}{G}\frac{d\ln\rho}{d\ln r}\right).$$

**Observational Data and Fitting:** Applied gas density and double-region temperature profiles across sample galaxy clusters (including A133, A383, A478, A907, A1413, A1795, A1991, A2029, A2390, MKW4). Linear Regression ($M = M_{hyd}/M_{obs}$), reduced chi-squared ($\chi^2_\nu$), and Gaussian relative likelihoods were calculated.

## Discussion & Result

**Scenario 1**: **Absence of Dark Matter (vs Observed Baryonic Mass)**

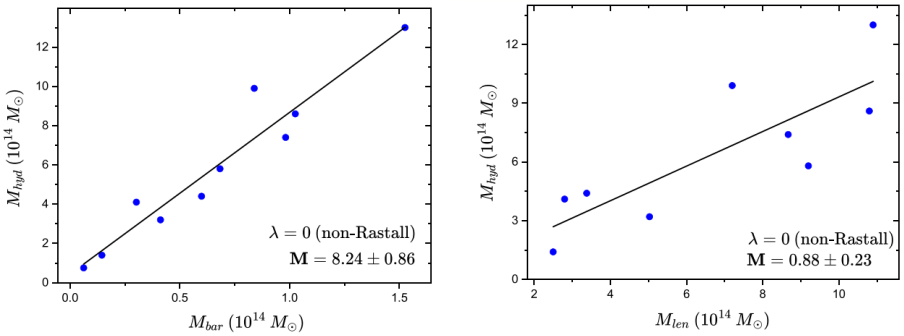
- Standard Non-Rastall Gravity ($\lambda = 0$): Yields a slope of $M = 8.24 \pm 0.86$, severely overestimating the mass.
- **Rastall Gravity** ($\lambda = 1.14\times10^{-1}$)**:** Yields a best-fit slope of $M = 1.07 \pm 0.1$, placing the ideal value $M = 1$ well within uncertainty and the peak likelihood distribution.

**Scenario 2: Presence of Dark Matter (vs. Observed Lensing Mass)**

- Standard Non-Rastall Gravity ($\lambda = 0$): Yields a slope of $M = 0.88 \pm 0.23$.
- **Rastall Gravity** ($\lambda = 1.29\times10^{-3}$)**:** Yields a best-fit slope of $M = 0.99 \pm 0.26$, successfully aligning the hydrostatic and lensing mass estimates and resolving the hydrostatic mass bias.

**Statistical Goodnes of Fit:**

- While Rastall gravity captures global scaling trends ($M \approx 1$), $\chi^2_{nu}$ analysis shows it does not universally outperform models such as non-local gravity when accounting for individual data point scatter.




*Picture 1. Linear fit to the relation between the hydrostatic mass according to standard non-Rastall gravity and the observed baryonic masses (left) and the observed lensing masses (right)*

## Conclusion

- Rastall gravity offers a viable phenomenological modification of gravity at cluster scales.
- It brings hydrostatic mass into direct agreement with baryonic mass without dark matter ($M = 1.07 \pm 0.11$) and resolves the hydrostatic mass bias when dark matter is present ($M = 0.99 \pm 0.26$).
- Expanding the cluster sample size in future studies will help establish tighter observational bounds on the Rastall parameter $\lambda$.