MODAL VIBRATION AND INTERNAL RESONANCES IN AN ELECTRICALLY ACTUATED SQUARE MICROMEMBRANE

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Abstract

We consider an initial-boundary value problem for a square micromembrane which is actuated by a DC voltage and an AC voltage. On the boundary the membrane is assumed to satisfy Dirichlet conditions. By using a multiple time-scales perturbation method, we describe the interaction between oscillation modes. It will be shown that if initially only odd modes are present, then those modes only interact up to $O(\varepsilon^2)$ with odd modes for times of $O(1/\varepsilon)$, where ε is a small parameter. On longer time-scales, there can be an interaction between odd and even modes. We also consider pure, superharmonic, and subharmonic resonance frequencies are obtained which are accurate up to $O(\varepsilon^2)$. The presented analytical results are in full agreement with the numerical simulations, which are obtained by using a finite difference method.

Keyword: MEMS, NEMS, MEMS, NEMS, Electrically Actuated Square Membrane-Modal Vibration-Resonance Frequency

Introduction

The role of Micro-electromechanical Systems (MEMS) and Nano-electromechanical Systems (NEMS) in all kinds of applications has significantly increased over the last decades. Due to its very high sensitivity, M/NEMS have been widely used in physical, acoustic, and gas sensors. Also, the low energy consumption, the reprogrammability of M/NEMS units to perform multiple logic operations, and the potential to operate under harsh environmental conditions, make electrostatic actuated M/NEMS resonators, an alternative in computing technologies which are using logic devices. Thus, it is essential to study and to understand the dynamics of M/NEMS, especially near resonance frequencies.

This research discuss the dynamics of the electrically actuated square micromembrane subject to Dirichlet boundary conditions. Unlike previous studies, for instance the semi-analytical approach by using the ROM method, we do not truncate the solution to a finite number of oscillation modes. Although the ROM-technique is beneficial due to the finite-dimensional system that one eventually obtains, it is evident that truncation of the solution to a few modes is not always applicable. In this research we consider the interaction between odd and even oscillation modes, and also we study resonance frequencies for some specific cases in this problem. As is well-known the resonance frequencies play an important role in the dynamics of M/NEMS resonators.

Research Method

The initial-boundary value problem of the scaled deflection of the membrane in the z direction at time t and at coordinate (x, y) is given by $w = w(x, y, t)$, which satisfies

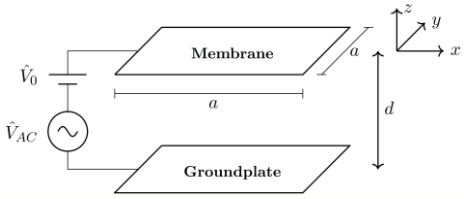
$$\frac{\partial^2 w}{\partial t^2} + \varepsilon^2 c \frac{\partial w}{\partial t} = \Delta w + \frac{\alpha (V_0 + \varepsilon \sin(\omega \tau))^2}{2(1 - \varepsilon w)^2},$$

$$0 < t, 0 < x, y < 1,$$

$$w(0, y, t) = w(a, y, t) = w(x, 0, t) = w(x, a, t) = 0,$$

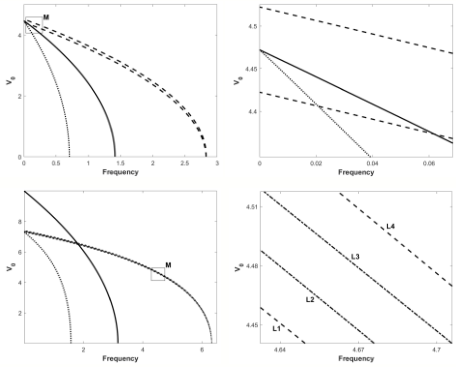
$$w(x, y, 0) = f(x, y), w_t(x, y, 0) = g(x, y),$$

where Δ is the Laplace operator, c is a constant related to damping coefficient, α is a constant related to the electric actuation, and ε is a small parameter. Here, the AC voltage V_{AC} is relatively small compared to DC voltage V_0.



Picture 1. *Schematic impression of the fixed groundplate and the actuated square micromembrane, as well as the electrostatic actuation $\hat{V}_0 + \hat{V}_{AC} \sin(\omega t)$, and the gap d between the actuated membrane and the groundplate.*

We use multiple time-scales perturbation method to analyze this initial-boundary value problem. Firstly, we find the equilibrium solution and look at the small solution near the equilibrium solution. Eventually, there are three cases to be considered related to the resonance frequencies, that is the pure, superharmonic, and the subharmonic. There are lots of number theory problems involved when we consider the interaction between modes in the $O(\varepsilon^2)$ problem. Since there can be lots of internal resonances, we just consider some cases, that is the single odd mode (1,1) that has no other interactions with other modes, and odd modes (1,3) and (3,1) that are interacting one with another. We compare our analysis with the numerical results we obtain using Finite Difference Method.



Picture 2*. The dependence of resonance frequencies on V_0. The right picture is the magnified box in the left picture. The upper pictures are for mode* (1,1), while the lower ones are for the interaction between mode (1,3) and (3,1)

Discussion & Result

By using the multiple time-scales perturbation method, we found the resonance frequencies that are essential in the study of the vibration of micromembrane. Since there are lots of internal resonances involved as we have proven in this research, we only consider some cases, that is finding resonance frequencies when single mode (1,1) is involved and the interaction between modes (1,3) and (3,1). In these two cases we determined completely the resonance frequencies accurate up to $O(\varepsilon^2)$, that depends on the DC voltage V_0 as depicted in the **Picture** 2. For each case, we consider the pure, superharmonic, and subharmonic resonance frequencies. The solid line is the point where we have the pure resonance frequencies, while the dotted line is the point where we have the superharmonic resonance frequencies. The domain between the dashed line are the points where we have the subharmonic resonance frequencies.

The work on this actuated membrane problem is far from complete. One can also consider other frequency domains for the actuation frequency, and other shapes for the membranes. The nonlinear membrane and plate problems are interesting extensions of the problem studied in this research.

Conclusion

We consider an initial-boundary value problem for an electrically actuated, square micromembrane subject to Dirichlet boundary conditions. By using eigenfunction expansions and the multiple time-scales perturbation method, we analyzed the interaction between oscillation modes. We gave an analysis that for this kind of problem, the odd modes don't interact with even modes. On longer time scales, we can have interaction between the even and odd modes. Moreover, we also give the resonance frequencies that are accurate up to $O(\varepsilon^2)$ for some special cases.

References

E. Harjanto, W.T. van Horssen, and J.M. Tuwankotta, "On resonances in a weakly nonlinear microbeam due to an electric actuation", *Nonlinear Dynamics,* vol 104, no. 4, pp. 3157-3185, 2021.

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