Development of strong 3-rainbow coloring in graphs

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Abstract

Rainbow coloring is a concept in graph theory that ensures connectivity through paths with distinct edge colors. This research investigates strong 3-rainbow coloring, which connects every three vertices through rainbow Steiner tree, and extends this concept by introducing rigid strong 3-rainbow coloring, where the connecting structure is restricted to a rainbow Steiner path. Through theoretical analysis, mathematical proofs, coloring constructions, and computational verification, we establish its formal definition, derive bounds, and determine exact values for certain graph classes. The research has produced two scientific articles, one published and one submitted, contributing to the development of rainbow coloring theory.

Keyword: rainbow coloring, rainbow Steiner path, rainbow Steiner tree, strong 3-rainbow index, rigid strong 3-rainbow index.

Introduction

Rainbow coloring [1] is an edge-coloring of a graph that ensures connectivity through paths with distinct edge colors. This concept has been extended to *strong 3-rainbow coloring* [2], which requires every set of three vertices to be connected by a minimum-size tree whose edges have distinct colors. The minimum number of colors required is called the *strong 3-rainbow index*, denoted by $\mathrm{srx}_3(G)$. This concept is motivated by secure and efficient group communication, where vertices represent agents, edges represent communication links, and edge colors represent different passwords. Using minimum-size connecting trees helps reduce communication costs.

However, these connecting trees may contain branches, which are not suitable for communication systems requiring information to travel sequentially along a single path. This motivates the development of *rigid strong 3-rainbow coloring* [3], an extension of strong 3-rainbow coloring that requires every set of three vertices to be connected by a minimum-size rainbow path without branching. The minimum number of colors required is called *the rigid strong 3-rainbow index,* denoted by $\mathrm{rsrx}_3(G)$. This research aims to investigate the strong 3-rainbow index of graph operations, introduce the concept of rigid strong 3-rainbow coloring, and determine bounds and exact values of the rigid strong 3-rainbow index for certain graph classes.

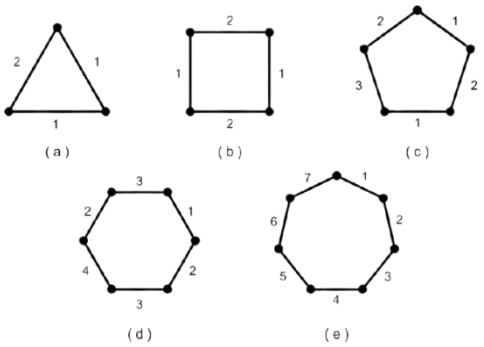
Research Method

This study employs a theoretical and analytical approach, combining structural analysis, combinatorial arguments, and constructive edge-colorings to investigate the strong and rigid strong 3-rainbow indices of graphs. First, lower and upper bounds are established through mathematical proofs based on graph properties and structures. Explicit edge-coloring constructions are then developed to demonstrate the attainability of these bounds, allowing exact values to be determined for certain graph classes and operations.

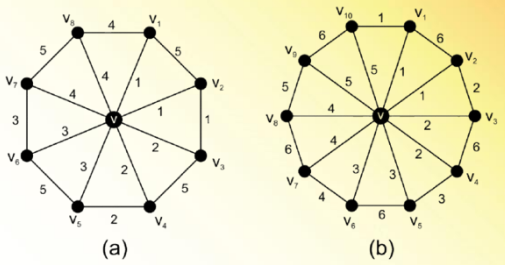
Discussion & Result

This research provides several theoretical results on the strong 3-rainbow index of graph operations and introduces its extension, the rigid strong 3-rainbow index. The main findings are summarized as follows:

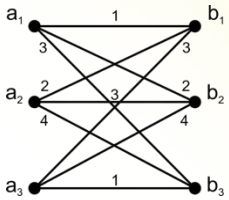
- Strong 3-rainbow index of graph operations: a sharp upper bound for the strong 3-rainbow index of corona product graphs was established, along with a characterization of graphs attaining this bound [4].
- Development of rigid strong 3-rainbow coloring: a new concept of rigid strong 3-rainbow coloring was introduced, including its formal definition and fundamental properties [3].
- Bounds and exact values: general lower and upper bounds were established, and exact values of the rigid strong 3-rainbow index were determined for certain graph classes through mathematical proofs and explicit coloring constructions [3].



Picture 1. Rigid strong 3-rainbow colorings of C_3, C_4, C_5, C_6, and C_7.



Picture 2. Rigid strong 3-rainbow colorings of W_8, and W_{10}.



Picture 3. A rigid strong 3-rainbow coloring of $K_{3,3}$.

Conclusion

This research advances the study of strong 3-rainbow coloring by establishing a sharp upper bound for corona product graphs and introducing the new concept of rigid strong 3-rainbow coloring. General bounds and exact values were obtained for certain graph classes, supported by mathematical proofs and coloring constructions. These findings contribute to the development of rainbow coloring theory and provide a foundation for further research on graph classes and operations.

References

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Output

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