Holographic Timelike Entanglement Entropy and Subregion Complexity with Scalar Hair

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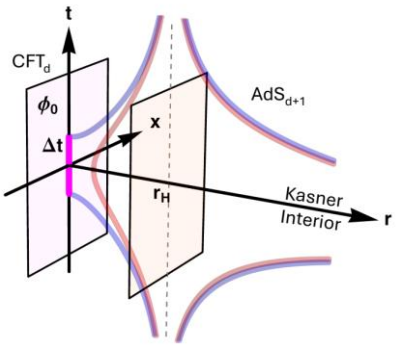
Abstract

We investigate the **holographic timelike entanglement entropy (HTEE)** and **timelike subregion complexity** of a thermal CFT_d deformed by a relevant scalar operator ϕ_0, dual to a hairy black hole in AdS_{d+1} by employing **the surface-merging prescription** (merging spacelike and timelike surfaces near the Kasner singularity at $r \to \infty$). We show that the relevant deformation breaks the invariance of $\mathrm{Im}[\mathrm{Area}]$ observed in pire AdS_3/BTZ , introducing a non-trivial time interval Δt dependence. Furthermore, analytic continuation of CFT temporal entanglement entropy fails to reproduce HTEE under boundary deformation. Additionally, timelike subregion complexity (ΔV) remains strictly real-valued, providing a complementary geometric probe of the black hole interior.

Keyword: AdS-CFT Correspondence, Black Holes

Introduction

Probing spacetime behind horizons remains a central challenge in gravitational physics. Relevant scalar hair deforms the bulk interior into a Kasner universe near the singularity ($r \to \infty$). HTEE and timelike subregion complexity are anchored to boundary subregions T with time interval Δt. To what extent do timelike extermal surfaces serve as reliable geometric duals of analytically continued CFT entanglement, and how do the probe trans-IR Kasner interiors under relevant boundary scalar deformations ϕ_0?



Picture 1. Ilustration of the AdSd+1 black brane setup

Research Method

Einstein-Scalar Gravity Action: The AdS_{d+1} black hole solution with massive scalar hair $\phi(r)$ with mass m in the context of holographic renormalization group is given by action function:

$$S = \frac{1}{\kappa^2}\int d^{d+1}x\sqrt{-g}\left(R - 2\Lambda - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{2}m^2\phi^2\right)$$

The metric ansatz for this hairy black hole is given by,

$$ds^2 = \frac{L^2}{r^2}\left(-fe^{-\chi}dt^2 + \frac{dr^2}{f} + dx^2 + d\mathbf{y}_{d-2}^2\right)$$

Surface-Merging Prescription: Extremal surface profiles satisfy:

$$t'^2(r) = \frac{K^2 r^{2d-2}}{f^2 e^{-\chi}}\frac{1}{fe^{-\chi} + K^2 r^{2d-2}},$$

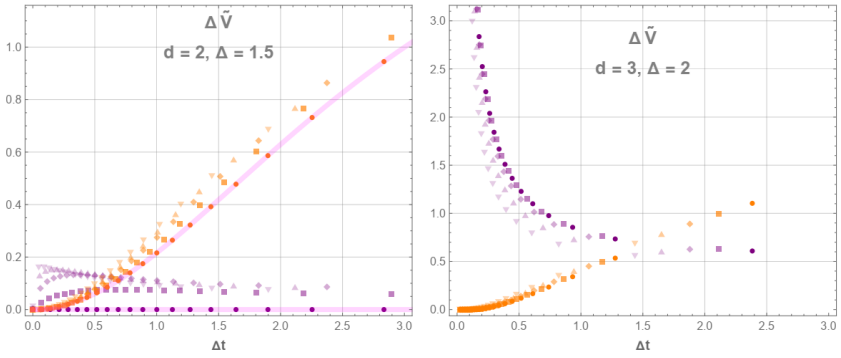
Where $d = +1$ corresponds to spacelike surfaces (real area) and $d = -1$ corresponds to timelike surfaces (purely imaginary area), which smoothly join near the Kasner singularity at $r \to \infty$.

Timelike Subregion Complexity (Complexity = Volume): Calculated within the CV proposal as the finite volume difference enclosed by $t_+(r)$ and $t_-(r)$ surfaces,

$$\Delta V \equiv V_{\mathcal{T}} - \frac{V_{d-2}L^d\Delta t}{(d-1)\varepsilon^{d-1}} = \frac{2V_{d-2}L^d}{d-1}\left[\int_\varepsilon^\infty \frac{t'_+(r)}{r^{d-1}}dr - \int_{r_0}^\infty \frac{t'_-(r)}{r^{d-1}}dr\right].$$

Discussion & Result

Smooth Surface Merging in Kasner Interior: Near the singularity ($r \to \infty$), $(t_d)^2 \to 0$ for both $d = \pm 1$, confirming that spacelike and timelike surfaces always **merge smoothly** regardless of scalar hair strength. Relevant scalar deformation **breaks the analytic equivalence** between spacelike and timelike entanglement entropy, introducing explicit time-interval dependence to the imaginary area component and causing standard CFT analytic continuation to fail. Meanwhile, holographic timelike subregion remains **strictly real-valued**, exhibiting **late-time linear growth** that is primarily governed by the interior volume behind the black hole horizon.



Picture 2. Plot comparing exterior vs interior contributions to $\mathrm{Im}[\mathrm{Area}]$ and timelike subregion complexity ΔV as functions of Δt across various scalar deformation strengths ϕ_0.

Conclusion

Independent Trans-IR Probe: Holographic timelike observables (HTEE and timelike complexity) encode critical black hole interior information that cannot be accessed from spacelike observables alone.

Broken Analytic Equivalenve: Relevant scalar operators reshape both exterior and interior bulk dynamics, destroying the analytic continuation bridge between spacelike and timelike entanglement.